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Lyakhova, S. & Neate, A. (in press). Further Mathematics, student choice and transition to university: Part 1 - Mathematics degrees. *Teaching Mathematics and its Applications: Sect A*

Further Mathematics, student choice and transition to university: Part 1 - Mathematics degrees

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Abstract

The transition from studying mathematics at school to university is known to be challenging for students. Given the desire to increase participation in science, technology, engineering and mathematics (STEM) subjects at degree level, it is important to ensure that the school mathematics curriculum is providing suitable preparation for the challenges ahead, and yet remains both accessible and popular. This two-part study investigates student choices of studying the post-16 A-level Mathematics and Further Mathematics qualifications in the UK, and their impact on the transition from school to university mathematics. Student opinions were accessed via a survey of undergraduate students and also individual interviews. This first part of the study considers the responses of mathematics undergraduate students and finds that both those who studied Further Mathematics and those that did not, perceive studying Further Mathematics as advantageous for their degree courses. However, the advantages identified mostly relate to the familiarity with topics, while students still feel unprepared for studying more abstract and proof-based mathematics. The study found that some factors which may be beneficial for transition currently lie outside the mainstream school mathematics syllabus and include studying through blended learning provided by the Further Maths Support Programme, practicing more advanced extension exam papers, and attending university outreach events. The choice of Further Mathematics is found to be influenced by the attitudes of the students, their teachers and their parents, to both mathematics as a subject, and to Further Mathematics as a qualification as well as student perceptions of Further Mathematics and their plans in terms of degree and university choice.

1. Introduction

Students need a strong foundation of mathematical skills to make a successful transition from school to studying science, technology, engineering and mathematics (STEM) disciplines at university. However, the dual nature of mathematics as both a subject in its own right and a tool for use in other disciplines, creates difficulties in designing a suitable school curriculum which balances these often conflicting desires. The aim of increasing participation in STEM subjects in the UK has also led to a need to broaden the appeal of mathematics courses at schools. It has been observed for some time that in attempting this, the depth of the mathematical content of post-16 qualifications has been gradually replaced by a breadth of material, which has weakened the ability of these qualifications to suitably prepare students for studying highly mathematical subjects at university (Hoyles, Newman and Noss, 2001). It is now acknowledged that schools alone cannot resolve these issues and universities have been urged to put in place appropriate support mechanisms to ease student transition (Hodgen, McAlinden and Tomei, 2014; Smith, 2017).

In England and Wales, the study of mathematics remains compulsory only up to the age of 16 when students take General Certificate of Secondary Education (GCSE) examinations in a range of subjects. Between the ages of 16 and 18, students who are intending to apply to university most commonly study for General Certificate of Education Advanced level (A-level) qualifications with students typically choosing to study for qualifications in 3 or 4 subject areas. Since 2000, all A-levels have had a modular structure, and students have been able to receive an Advanced Subsidiary level (AS-level) qualification for completing half of the A-level modules. There are currently two mathematics A-level qualifications available: 'Mathematics' and the more advanced 'Further Mathematics' which can only be studied in addition to 'Mathematics'. There are also additional exam papers that students taking the mathematics A-levels can choose to sit, such as the Sixth Term Examination Paper (STEP) or the Maths

Admissions Test (MAT). These exams attempt to test the A-level material in greater depth and are required for entry by some competitive UK universities to differentiate between the best performing students.

Further Mathematics is seen as a good or better preparation for mathematically demanding university degrees (Bowyer & Darlington, 2016; Darlington, 2015a; Darlington & Bowyer, 2016a; Darlington & Bowyer, 2016b) and is one of the "facilitating subjects" recommended by the Russell Group of leading UK universities (Russell Group, 2016). It has been repeatedly suggested that universities should encourage prospective students to take more mathematics courses such as Further Mathematics when applying for degrees with rich mathematical content (Morgan, 2011; Porkess, 2008; Hodgen, McAlinden & Tomei, 2014; Smith, 2017) with the positive influence on student transition often attributed to the overlap in their content (ACME, 2011). However, this is not often reflected in university entry requirements, and only 9% of mathematics undergraduate degree courses in the UK required Further Mathematics for entry in 2016 (Lord & Lee, 2017).

However, for several years, numerous shortcomings in the structure and content of both mathematics A-levels have been noted. Their modular structure, their increasingly compartmentalised approach to learning and their rigid mark schemes have been repeatedly criticised (Tavener, 1997; Bassett et al, 2009) and can be seen as encouraging students to adopt the surface rather than deep approach to learning.

Surface and deep approaches to learning were defined by Martin & Saljo (1976) and later developed by Biggs (1993). The surface approach to learning is typically an attempt to meet the requirements of a course or assessment with the least effort, adopting strategies such as rote learning. In contrast, a deep approach to learning is one in which a student engages at a meaningful level with the material to better understand a subject, and it is this approach which is widely expected of students in Higher Education. Indeed, this contrast in the pedagogies and assessment practices between the mathematics A-levels and undergraduate courses has been associated with making the transition from school to university more challenging (Darlington, 2014).

Given this background, it is not surprising that all A-level qualifications have recently been the subject of a government reform process which has revised their content and also changed their structure from modular to linear, with students beginning to study for the new mathematics A-levels in September 2017. Indeed, the transition from school to undergraduate mathematics is a focus in the ongoing reforms of both A-levels and mathematics education in England and Wales in general (see e.g., ALCAB, 2014; Smith, 2017).

This is the first paper in a two-part study of undergraduate students and Further Mathematics and follows on from an earlier study of school children (Tanner, Lyakhova & Neate, 2016). Here, we study the opinions of mathematics undergraduate students, accessed through questionnaires and individual interviews, on their choice of A-level Further Mathematics (as studied before the new reforms were introduced in September 2017) and the impact of this decision on their transition to studying for a mathematics degree. More generally, we are concerned with how students of STEM degrees make their A-level and degree choices and whether Further Mathematics plays any specific role in this. A further study of students studying for a non-mathematics STEM degree will be published separately.

2. Participation in Further Mathematics

At present Mathematics is the most popular A-level with 11.5% of all A-level students awarded the qualification in 2017, while Further Mathematics is one of the least popular options with only 1.8% of students studying the subject in 2017 (JCQ, 2017). However, the current uptake is a substantial improvement on the participation rates a decade ago (see Tanner, Lyakhova

& Neate (2016) for selected data). Participation in Further Mathematics has been restricted in part due to a lack of qualified teachers, timetabling issues and financial constraints in schools and colleges. To address some of these difficulties, the Further Mathematics Support Programme (FMSP) was established in 2003 in England and in 2010 in Wales. The FMSP provides an opportunity for students to study Further Mathematics via blended learning using online and face-to-face tuition when it is not offered by their school or college, and also supports provision in schools by offering professional development for teachers. While only a small proportion of all Further Mathematics students take their courses with the FMSP (approximately 2% of students awarded A-level Further Mathematics in the UK in 2017 studied with the FMSP (JCQ, 2017)), the uptake of the subject in England and Wales since its inception has been significant. The proportion of A-level Mathematics students who were also awarded A-level Further Mathematics increased from 11% in 2003 to 17% in 2016 in England and from 7% in 2010 to 13.5% in 2016 in Wales.

In examining the general increase in participation in both mathematics A-levels, Noyes & Adkins (2016), noted a "pipeline" of high achieving students at GCSE Mathematics proceeding to study A-level mathematics and suggested that the improvements in participation rates in both A-level Mathematics and Further Mathematics could be the result of improvements in GCSE grades. However, this "pipeline" could also be seen to restrict participation to only those who had previously excelled at mathematics. This effect could be exacerbated in Further Mathematics in part due to its perception amongst students as the most challenging A-level subject (Hillman, 2014) and also because the FMSP approach to tuition expects a lot from students in terms of workload and self-study. To address this, it has been argued that whenever possible students should be offered Further Mathematics as a fully timetabled option in their school (see Tanner, Lyakhova & Neate (2016) for a discussion).

The principal factors reported to influence Further Mathematics choice include positive attitudes to mathematics as well as the intention to take a degree with high mathematical content (Darlington, 2015a; Bowyer & Darlington, 2016; Tanner, Lyakhova & Neate, 2016). But a lack of role models and a lack of an appreciation of the qualification amongst parents have been observed as factors limiting participation (Tanner, Lyakhova & Neate, 2016). Lord & Lee (2017) state that while 64% of all students in England took three A-level subjects in the 2016/17 academic year, 52% of Further Mathematics students took four or more subjects, implying that Further Mathematics is often their "fourth subject" taken in addition to their other subject choices.

While female students are three times less likely to study Further Mathematics than male students (Lord and Lee, 2017), little research has been conducted specifically about girls participation in Further Mathematics. In general terms, good practices for encouraging more girls to study mathematics post-16 have been recognised as those that benefit all students (Baldwin, de Pomerai & Smith, 2016; Smith 2014; Smith & Golding, 2015). Specifically relating to Further Mathematics, Tanner, Lyakhova & Neate (2016) identified a few gender differences amongst students studying AS/A-level Mathematics, namely that girls were less likely to consider Further Mathematics more difficult than A-level Mathematics, that only the gifted should study Further Mathematics, and that Further Mathematics provides a route to a better job. Darlington & Bowyer (2016b) noted that among undergraduate chemistry students, girls were more likely to have chosen Further Mathematics if they believed that mathematics was their best subject but boys were more likely to have been influenced by how interesting the topics in the Further Mathematics course appeared. Some of these differences may be related to evidence that girls attribute their success in mathematics to effort rather than luck or ability (McLeod, 1992).

3. Methodology

Sample: In phase one of the study a survey was conducted of 377 undergraduate students from 7 universities across England and Wales who were studying for the first year of a STEM degree. From these students, 367 who had studied at least AS-level Mathematics were selected for analysis.

In this paper, we consider the responses of the 236 mathematics undergraduate students in this sample (61% male, 39% female). All these students had studied for A-level Mathematics and 138 had also studied AS-level Further Mathematics (58% male, 42% female) with 102 of these also completing A-level Further Mathematics (55% male, 45% female). The students had almost all performed well in both GCSE and A-level Mathematics, but there is a range of performances in Further Mathematics (see Table 1). The sample had a higher proportion of girls studying Further Mathematics than in the UK (JCQ, 2017). From those taking Further Mathematics, 9 had done so through the FMSP.

Table 1. Mathematics undergraduate student sample by school exam results with national results for comparison (JCQ, 2017).

Qualification		A*	A	B	C	D	E	U
A-level Further Maths	Sample	19%	31%	23%	19%	8%	1%	0%
	UK	30%	28%	19%	11%	7%	3%	2%
A-level Maths	Sample	29%	41%	25%	4%	1%	0%	0%
	UK	18%	24%	22%	16%	11%	6%	3%
GCSE Maths	Sample	66%	29%	4%	0%	0%	0%	0%[1]
	UK	2%	3%	6%	22%	29%	19%	20%[1]

[1] Includes grades F,G,U.

The sample was selected based on university entrance standards in mathematics. Universities across England and Wales were categorised into three groups based on their mean entry standard for mathematics degrees (taken from the Complete University Guide 2017), as detailed in Table 2. Entry standards were measured using the University and College Admissions Service (UCAS) qualification points system which allocated 140 points for an A-level at grade A*, 120 points for a grade A, down to 40 points for a grade E. For reference, of the 61 universities in England and Wales classified for mathematics in the Complete University Guide 2017, 9 fell into Group 1, 24 into Group 2 and 28 into Group 3.

Table 2. University Group classifications and the mathematics undergraduate survey sample.

University Group	*Mean entry level for mathematics degrees:*		*Survey participants:*			
	UCAS points	*Equivalent A-level grades*	*Total*	*% female students*	*AS Further Maths*	*A-level Further Maths*
Group 1	*>480*	*Better than AAAA*	*54*	*40%*	*50*	*46*
Group 2	*340 – 480*	*AAB – AAAA*	*122*	*43%*	*66*	*44*
Group 3	*<340*	*Below AAB*	*60*	*29%*	*22*	*12*

Participating universities were selected to cover the three university groups. A named contact at each university who had agreed to assist with the survey was sent paper surveys with instructions to distribute them in a lecture of first year undergraduates. It was requested if possible that students were given time in the lecture to complete the survey to avoid a self-selecting sample. All participating universities followed this procedure with one exception where students were requested to complete the survey in their own time. This university had a much lower participation rate. It should be noted that none of the universities involved in the survey required students to take Further Mathematics for admission to their undergraduate mathematics degrees.

Results described in Sections 4.1 and 4.2 are based on these 236 mathematics undergraduate students. In Section 4.3, we discuss participation in teaching delivered by FMSP. Due to the small numbers involved, we consider the responses of the 173 students who had studied Further Mathematics taken from the 367 students in the full survey, which includes students studying for both mathematics and non-mathematics STEM degrees. In total, there were 18 students in this sample who had studied with the FMSP.

In phase two, 14 students (7 female and 7 male) from 7 universities who were studying for a STEM degree were interviewed. From these, the responses of 8 students (4 male and 4 female) who were studying for a Mathematics degree, are considered in this paper. Initially, students from the survey sample (who had indicated they would be happy to be contacted) were approached for interviews, and four agreed to participate. As this initial response was poor, more students from the same universities were approached as well as students who had interacted with FMSP Wales during the previous year, with the aim to achieve a balance across university groups and gender. Consequently, not all the universities involved in phase two were involved in phase one. The sample of mathematics undergraduates involved in phase two contained 4 students from Group 1 universities, 1 student from Group 2 and 3 from Group 3. Amongst these students 6 had studied Further Mathematics at least to AS-level, 2 had studied through the FMSP and 4 had studied for STEP papers.

Ethical approval for both the use of the questionnaire and the interviews was sought and given prior to the commencement of the study. Free and informed written consent was sought from all participants. The research was conducted in accordance with BERA guidelines (BERA, 2011).

Instruments: The student questionnaire contained a series of statements (see Appendix 1) which covered several affective variables that were known to influence students' learning of mathematics (McLeod, 1992). These included attitudes to mathematics and beliefs about mathematics (Schoenfeld, 1985), mathematics self-concept (Marsh, 1986), previous experience of doing mathematics and teacher/parental influence (Mensah, Okyere & Kuranchie, 2013; Ma, 2001). There were also questions related to surface/deep learning approaches and student perceptions of the benefits of studying Further Mathematics. Out of the 37 questions across various categories, 12 related to mathematics as a subject, 10 to GCSE and A-level Mathematics, and 15 to A-level Further Mathematics. Some of the questions related to Further Mathematics have a different interpretation depending on whether a student had studied Further Mathematics or not. Each question used a 5 point Likert scale from strongly agree to strongly disagree.

The analysis of the interviews has been informed by the findings of the longitudinal study of Cleaves (2005) who investigated science A-level choices in high achieving students. This study showed that choices were formed as an interplay of several factors, such as self-perception, relationships with significant adults, and experiences of subjects both in school and outside of school, but also related to whether students felt resolved about their future career or degree choices.

4. Results

Where statistical comparisons are made between groups, non-parametric tests have been used as the Likert scale data from the survey is ordinal. Mann-Whitney tests have been used for comparisons between two groups and Kruskal-Wallis tests for those between three groups, both with significance level $\alpha=0.05$. The estimate of effect size 'r' is defined by $r=Z/\sqrt{N}$ where N denotes the sample size with effect sizes taken as "small" ($r=0.1$), "medium" ($r = 0.3$) and "large" ($r=0.5$) (Cohen, 1988). In this study, students who had studied for at least AS-level Further Mathematics were classified as having participated in Further Mathematics. Full tables of the results and statistics can be found in Appendix 2.

4.1. Student attitudes to mathematics and choice of Further Mathematics.

There was an almost universal positive attitude to mathematics among survey respondents, with 95% agreeing that they enjoyed mathematics and 73% planning to continue studying it to the highest possible level. This was underpinned by their previous success in mathematics with 92% and 74% respectively agreeing that they had found GCSE and AS-Level Mathematics easy.

There were several significant differences between those who had chosen to study Further Mathematics when compared with those who had not (see Figure 1 and Table 3). These show that studying Further Mathematics is associated with students being more confident in their own mathematical abilities, having higher aspirations about their future university and career and a stronger feeling that they were encouraged by teachers and parents. In interviews, all the students who had studied Further Mathematics emphasised how much they enjoyed mathematics, and some explained that they had valued Further Mathematics for providing them with the challenge that was missing from their A-level Mathematics classes. Several of the interviewed students had role models with a strong mathematical interest among their close family to whom they could relate and, more importantly, discuss their choice, but it was most commonly their teachers who had ultimately prompted them to take the qualification. In the survey, the student interpretation of parental attitudes towards Further Mathematics were not as favourable as the attitudes to Mathematics. In fact, 54% of all students were unsure how their parents felt about Further Mathematics and only 22% agreed that their parents thought it was important, whereas 55% of students felt that their parents agreed it was important to study for at least AS-level Mathematics.

There were only a few significant gender differences in student attitudes to mathematics and Further Mathematics identified in the survey which are shown in Figure 2 and Table 4. These imply that boys and girls can have differing mathematics self-concepts but also show that motivating factors can be different for boys and girls, with boys more likely to see Further Mathematics as a route to a good job.

For students who decided not to take Further Mathematics, there appears to be a complex range of factors that might have influenced this decision. For most of the students it does not appear to be a simple lack of belief in their own ability that influenced them, with only 4% agreeing that they were not good enough to study Further Mathematics (see Figure 3). However, there was a significant difference between the responses of boys and girls on this matter with girls less likely to disagree to this statement (see Figure 2 and Table 4 part a). Although students who studied Further Mathematics were significantly more likely to feel they were encouraged by their teachers in pursuing the subject, only a small proportion of those who did not take Further Mathematics felt they were discouraged by their teachers from studying it (see Figure 1 and Table 3). There is evidence of a stigma associated to Further Mathematics with 35% of those who did not take it agreeing that it was only suitable for those who are gifted at mathematics. However, there is no significant difference between those who did and those who did not take Further Mathematics on this matter ($p=0.34$, $Z=-0.96$, $r=-0.06$). Evidently, for some students studying for a breadth of subjects was more important than studying more mathematics, some perceived Further Mathematics as difficult or requiring more self-study and some did not see it as necessary for studying at a good university (see Figure 3).

This was supported at interview, where the choice of subjects made by the students was seen to be influenced by a variety of factors, and the strategies that they employed often depended on whether they had felt resolved about their future. For example, a student who "always knew" that he would study for a mathematics degree explained his A-level choices as resulting from a desire to pursue a broad range of subjects at school before he specialised at university. This student chose History in place of Further Mathematics aiming to improve his writing skills. He had previously studied an additional mathematics course alongside GCSE Mathematics which gave him an advantage over other students in his A-level Mathematics class. However, this experience had led him to feel that the benefits of such an extra qualification were short-lived and he had viewed Further Mathematics in the same way, believing "it would help but only up to a point". For him, studying History represented a higher utility value when compared to Further Mathematics. In addition to this, Further Mathematics was not offered directly in his school and he felt that "travelling to another school [to attend Further Mathematics classes] was a deterrent". This involved studying with an unfamiliar teacher, in an unfamiliar environment which he perceived would be more difficult than taking another subject at his own school.

The opinions of the respondents who had studied Further Mathematics supported some of the challenges perceived by those who did not take the subject. In the survey sample, 79% of students who had taken Further Mathematics agreed that there was a step-up in difficulty from studying A-level Mathematics and 60% agreed it required more self-study than other subjects. The interviewed students, both those who studied Further Mathematics in their school as well as those who studied through the FMSP, emphasised that it was the fast pace of the Further Mathematics classes that they found the most challenging. One of the respondents explained how he understood the reasons for this: "I guess the people who were studying Further Maths were a lot more interested in maths so [the class] went faster". This student further argued that it left little space for mastering mathematics in depth and it was "way harder to get an A [grade] in Further Maths than Maths".

Given these challenging aspects, one may assume that students who stuck with Further Mathematics were highly motivated in pursuing the subject, and indeed, 61% of these students felt that studying Further Mathematics was more important for them than studying a range of subjects. However, for some students, taking Further Mathematics was perfectly compatible with choosing a broad range of subjects. For instance, some interpreted taking a "broad range" to mean choosing Mathematics and Further Mathematics together with an "alternative" highly contrasting subject such as Art or Sport.

There is evidence that an aspiration to get into a "better university" may have influenced some students with a significant difference between university groups on this (see Figure 4 and Table 5 part a). Step-down analysis showed that there was a significant difference between students attending Group 1 universities compared with those who attended Groups 2 or 3 but with no significant difference between Groups 2 and 3 ($p=0.576$). This was reflected in interviews where students from university Group 1 emphasised that it was Further Mathematics that helped them to get their place in a "prestigious" university even though the universities involved did not require Further Mathematics for admission. Causality is difficult to establish as there were students in our interview sample who were initially directed to studying Further Mathematics by their teachers and only became fully aware of the benefits in terms of university applications later.

For some students who chose Further Mathematics, it was the combination of utility and enjoyment that was valued the most. For example, one student who was considering both mathematics degrees and courses with "some kind of numerical focus", valued Further Mathematics for helping him to both choose to take a mathematics degree and also secure a place in a good university. Initially he was motivated to take Further Mathematics by its perceived high exchange value, but later on the enjoyment of doing mathematics became more important for him. This student appreciated the opportunity to study Further Mathematics so much that he did not mind doing it through a combination of school and online classes. He summarised the overall impact studying Further Mathematics had on him: "Had I not taken extra mathematical units… I do not know whether I would still have made that decision [studying for a mathematics degree]. Without Maths and Further Maths, I would not be where I am today".

One student described how Further Mathematics helped shape her future degree and career choices after she had arrived at studying Further Mathematics "by chance" as other subjects did not fit her timetable. Initially she had been choosing between a future career as a mathematics teacher or an accountant, but once she began studying Further Mathematics she changed her view of what mathematics could offer her and dismissed her earlier choices as "immature". While both her earlier choices were examples of the so-called "high visibility" occupations, the choice of an accountant was strongly influenced by her parents' business.

As one may expect, the students who felt "unresolved" about their future were more active in soliciting help from adults and peers including getting advice from older students who were already studying for a mathematics degree. These students evidently felt pressurised to choose subjects that kept their options open. Reflecting on their choices, one student who had chosen only STEM subjects for their A-levels remarked: "If I had known at the time where I was headed, with hindsight I would have taken something that I simply had an interest in, such as English, replacing Chemistry".

Figure 1. Selected significant differences (Mann-Whitney) between those who had and had not studied Further Mathematics amongst Mathematics undergraduate students.

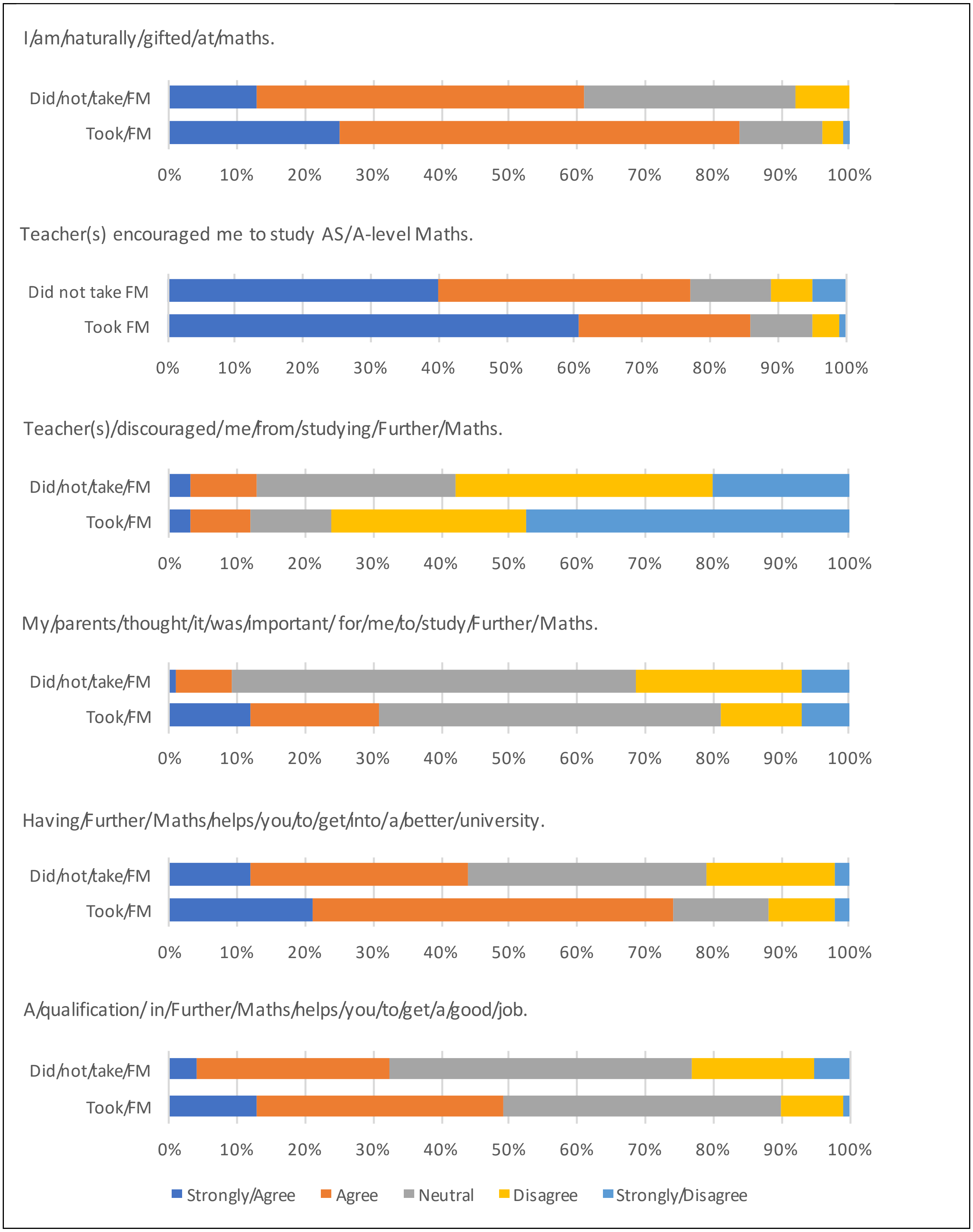
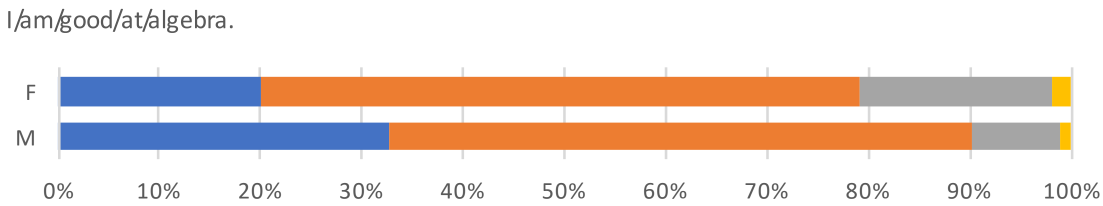
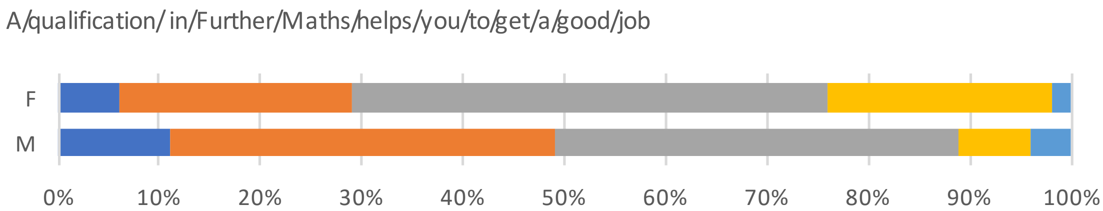


Figure 2. Significant differences (Mann-Whitney) between genders amongst Mathematics undergraduate students.

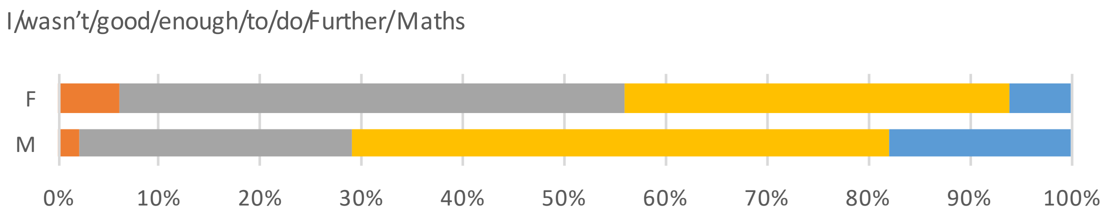
a) Attitudes to maths and Further Maths:

All Students:





Students who had not studied Further Maths:



b) Considering transition to and studying at university:

Students who had studied Further Maths:

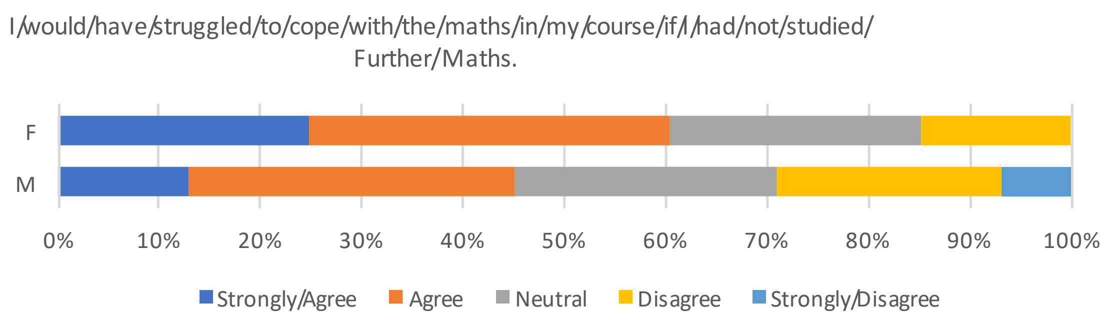


Figure 3. Mathematics undergraduate students who did not take Further Maths, reflecting on their motivations for not choosing it.

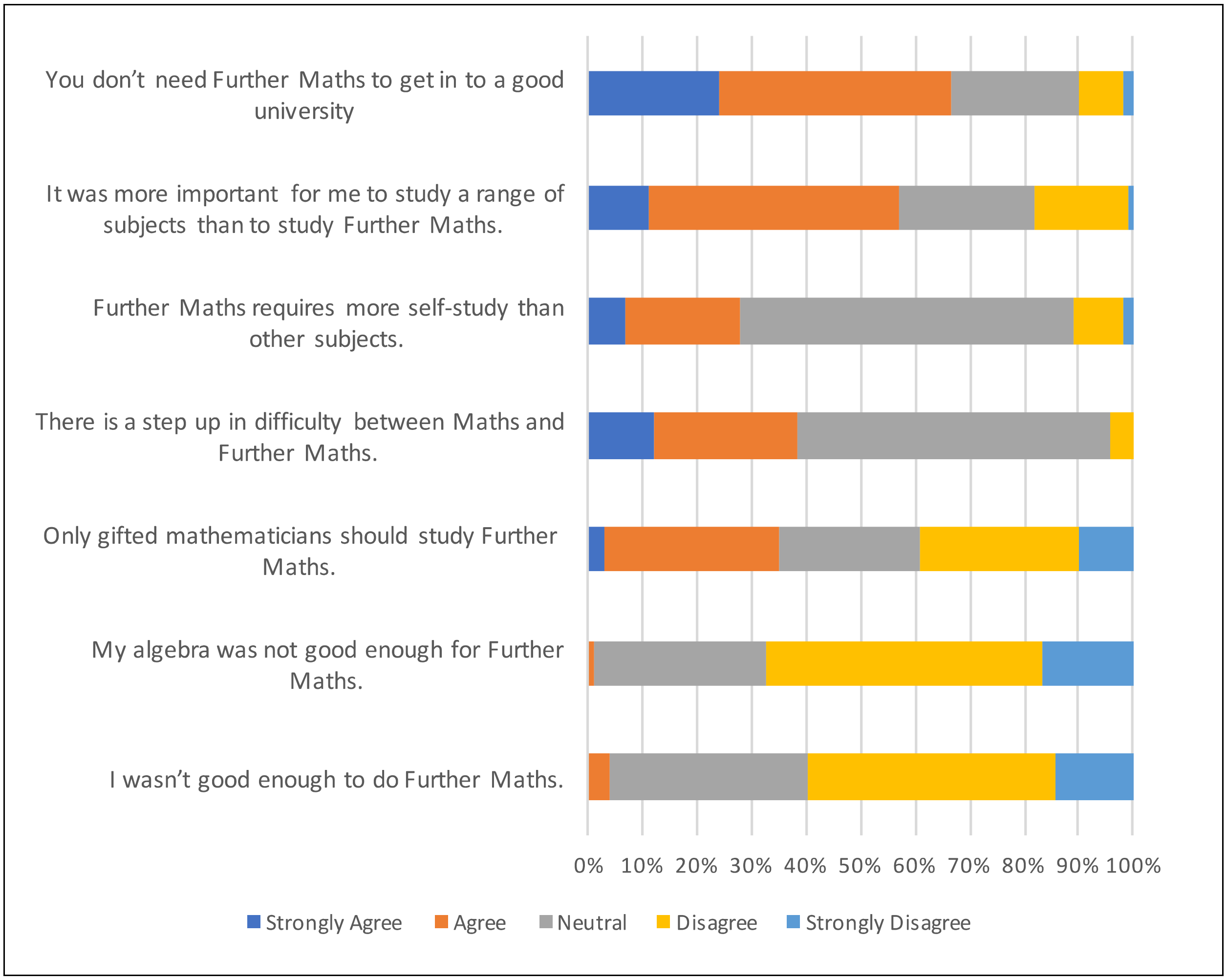
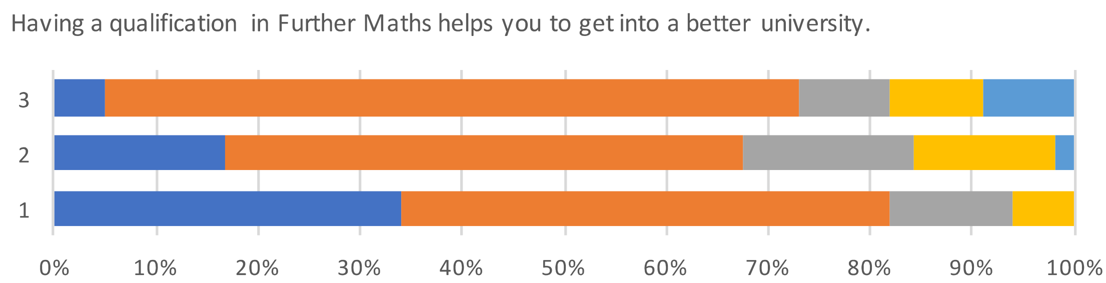


Figure 4. Selected significant differences (Kruskall-Wallis) between university groups[1] amongst Mathematics undergraduate students.

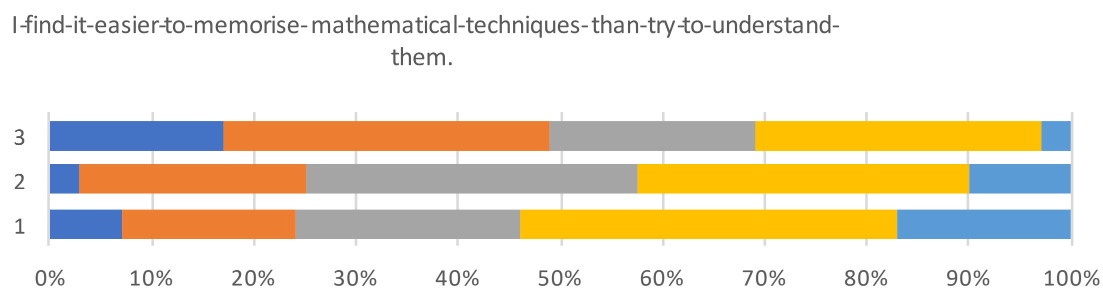
a) Attitudes to mathematics and Further Maths:

Students who had studied Further Maths:

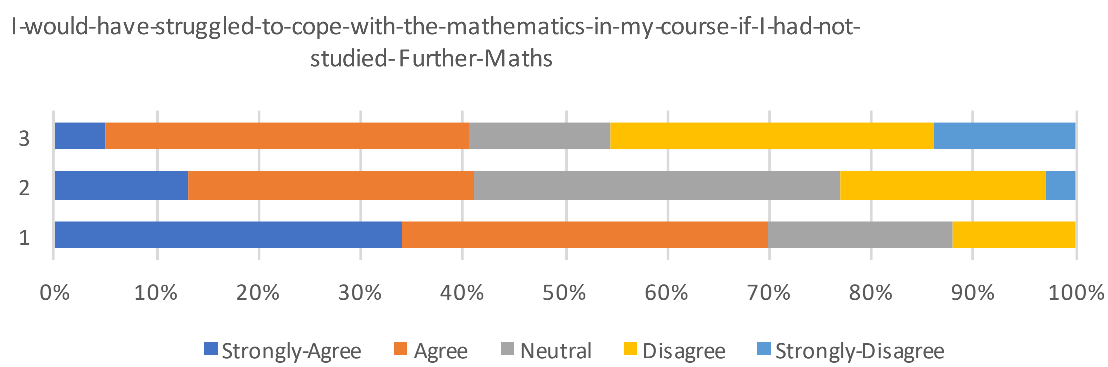


b) Considering transition to and studying at university:

All Students:



Students who had studied Further Maths:



[1]University Groups as defined in Table 1 for mean UCAS points on entry Group 1:>480, Group 2: 340-480, Group 3: <340.

4.2 Further Mathematics and Transition:

In the survey, 90% of those who studied Further Mathematics agreed that they felt it had made their transition to university easier with no significant difference in the responses across the university groups (Kruskall-Wallis, p=0.176, N=136). Moreover, 89% agreed that they perceived the qualification had given them a broader understanding of the subject in general. Of those who did not study Further Mathematics, 71% agreed that they now wished that they had done so, with no significant difference across the 3 university groups (Kruskall-Wallis, p=0.196, N=97). This suggests that Further Mathematics is perceived to be beneficial by mathematics undergraduates at a range of universities with differing entry requirements, not just by those attending the most selective universities. Indeed, in interviews, students across all university groups who had studied Further Mathematics were clear that they perceived the advantage to be in the knowledge of new facts learned on the course, in particular related to matrices, complex numbers, vectors and advanced integration and differentiation techniques. However, students also perceived that it helped more generally, for example one student commented that "just doing more maths helps leading into university…" with "…less worry about seeing new crazy stuff since [I] had seen it before". There is however evidence that those attending Group 1 universities perceive a greater direct utility from studying Further Mathematics, with a significant difference between university groups amongst those who studied Further Mathematics on whether they would have struggled without it. In pairwise analysis, students from Group 1 universities were more likely to feel that they would have struggled when compared with either Group 2 (p=0.004, Z=-3.21, r=0.25) or Group 3 (p=0.001, Z=-3.50, r=0.33), with no significant difference between Groups 2 and 3 (p=0.726, Z=-1.17, r=0.13), (see Figure 4 and Table 5 part b). There was also a significant gender difference here (see Figure 2 and Table 4 part b), with girls more likely to say that they would struggled to cope with their course without Further Mathematics than boys.

Yet there were also areas where there was no evidence that Further Mathematics had made an impact. When asked about whether they now have to work hard "just to keep up", there was no significant difference between those who had studied Further Mathematics and those who had not, (Mann-Whitney, p=0.06, Z=1.88, N=235, r=0.12) with 46% of all students agreeing that they have to work hard. There was also no evidence that studying Further Mathematics influenced the approach students adopted to studying mathematics, with 31% of all students agreeing that it was easier to memorise the material rather than to understand it and no significant difference in the survey responses between those who had and had not studied Further Mathematics (Mann-Whitney, p=0.078, Z=1.76, N=235, r=0.11). There is however evidence of a significant difference here when compared between university groups (see Figure 4 and Table 5 part b). In pairwise analysis, students attending a Group 3 university were more likely to say that memorising mathematical techniques was easier than developing understanding than students from either Group 1 (p=0.004, Z=3.22, r=0.30) or Group 2 (p=0.016, Z=2.78, r=0.20), with no significant difference between Groups 1 and 2 (p=0.946, Z=1.00, r=0.08). This was supported in the interviews where students from Group 3 universities emphasised how unprepared they felt for introductory courses in real analysis.

The failure of Further Mathematics to alter student approaches to mathematics was reflected in interviews where students often referred to focusing on practicing algorithmic procedures in both A-level Mathematics and Further Mathematics. This contrasted to their descriptions of the undergraduate mathematics as more abstract and proof-based with an emphasis on constructing valid mathematical arguments. One student, for example, described the mathematics he met in his university course as "very little computation and more thought" and remarked: "Whereas at A-level, it was possible to learn a particular method to get you through questions, at university you are expected to be able to think for yourself, and no single approach will allow you to tackle every question." While at A-level many examples are given to help to learn a particular method, "at university you would be lucky to get one". One of the respondents used the word "repetitive" to describe what A-level was and what university mathematics was not. Several students remarked upon how much faster paced the learning of mathematics was at university, which some (but not all) interpreted to mean that "you are required to put more work in yourself to understand a new concept".

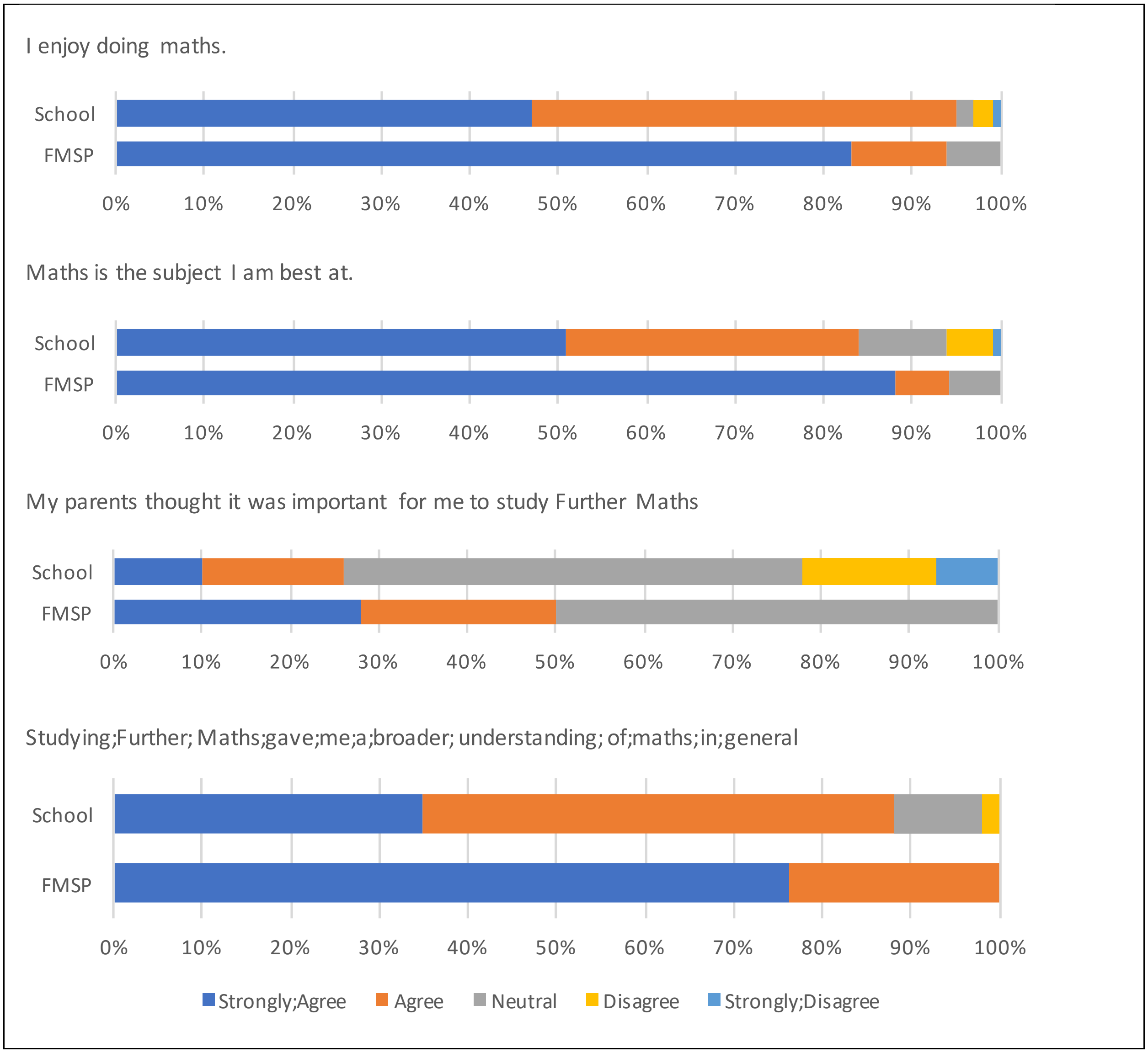
4.3. Experiences of mathematics outside school

There is evidence from the surveys and interviews that several factors which are not a formal part of the school mathematics curriculum influence student appreciation of mathematics as well as the transition to university.

The first factor identified was studying Further Mathematics through the FMSP. In the full survey sample of 376 students (including those who studied for a non-mathematics STEM degree), there were 173 students who had studied Further Mathematics, 18 of whom had studied through the FMSP. Here we consider this full sample of both mathematics and non-mathematics undergraduate students due to the small numbers studying through the FMSP. There are significant differences in the opinions of the FMSP students when compared to those that studied Further Mathematics in school (see Figure 5 and Table 6). Though the sample of the FMSP students is small, the differences are all small to medium sized effects. The FMSP students are more likely to enjoy mathematics, more likely to think of mathematics as their best subject and more likely to have parental support. But the most striking finding is that students who used the FMSP appeared to have perceived a greater gain from the experience as they are more likely to perceive that studying Further Mathematics had broadened their understanding of mathematics in general.

These differences can be interpreted as reflecting the nature of the FMSP experience which can involve studying several modules simultaneously, reduced contact teaching, being taught by different tutors, and having a large amount of self-study, all of which make the FMSP courses closer to the university learning experience for the students. One of the interviewed students, for example, explained the added value of the FMSP courses: "It also helps you be more organised and independent as an individual. When studying through the FMSP, you have to take a lot more responsibility for your own learning. The skills involved with doing this are incredibly useful in the transition [to university]."

Figure 5. Selected significant differences (Mann-Whitney) between those who studied Further Mathematics using the FMSP and those who studied it in school amongst STEM undergraduate students.



Another experience that was highlighted by students at interview was studying extension papers, such as STEP. One of the students summarised the difference between the A-level work and the STEP work as follows: "While Maths helped to some extent, I feel that Further Maths helped more in terms of the extra content that was covered, whereas STEP […] prepare you more in terms of actual problem solving ability." The students explained this as a consequence of the nature of the STEP questions. These are longer, much more challenging and require a lot more independent thought as well as the ability to link topics from different areas of the syllabus. For the students the main challenge was "thinking for yourself" which they did not see as essential to succeed at A-level. The students emphasised that while it was not necessary to perform well at these exams or even to take them for gaining a place on a mathematics degree course, they felt that "just having had that exposure will make it easier for you to adapt to the differences between A-level and University".

Finally, a few students also reflected on experiences such as summer schools, student conferences and festivals that helped them to see mathematics in a new light. In their opinions, learning as much mathematics beyond the syllabus as possible was the best preparation for university mathematics and "made less of a shock going into university".

5. Discussion and Conclusion

We have evidence in this study that students who go on to study for a mathematics degree perceive benefits from studying Further Mathematics. Amongst those who studied Further Mathematics we see this in their perception that they would have struggled without the qualification, and amongst those who did not study Further Mathematics we see that in retrospect many wish they had. The perceived advantage appears to lie mainly in the additional knowledge gained, but for some students there is evidence from the interviews that studying Further Mathematics may help clarify interests for those unsure about their degree choices or provide an additional challenge for those who do not feel sufficiently stretched in A-level Mathematics classes. Tanner, Lyakhova & Neate (2016) noted that studying Further Mathematics topics helps students to see how it is possible to push the boundaries of pure mathematics and it may be that this helps students see mathematics in new light which, in turn, may help to clarify their choice. Our findings suggest that those who attend the most selective universities perceive a greater utility from studying Further Mathematics than those attending other universities, which echoes previous findings (Bowyer & Darlington, 2016). However, we have seen that students at universities with lower entry requirements also perceive studying Further Mathematics as advantageous.

As with a previous study of school children studying for A-level Mathematics and Further Mathematics (Tanner, Lyakhova & Neate, 2016), the survey identified few gender differences. But it did identify some gender differences in how students perceived the utility of Further Mathematics, identifying that girls who studied Further Mathematics were sometimes more appreciative of its impact than boys.

However, the nature of this study provides us only with a snap shot of student opinions. It tells us about their retrospective attitudes and perceptions, rather than showing us how these developed in time. It also does not enable us to demonstrate an actual benefit from studying Further Mathematics in terms of improved outcomes or exam marks. However, a perception of benefit could be a positive outcome for some students, and so there is an argument to encourage wider participation in the Further Mathematics qualification.

Participation in Further Mathematics is not simply driven by previous successes in mathematics. In our sample, only 67% of those with an A* and 49% of those with an A at GCSE Mathematics had chosen to participate in Further Mathematics. The national picture is of an even lower participation rate in Further Mathematics; across all GCSE students in England, only 27% of those who achieved an A* in GCSE Mathematics in 2010 went on to study at least AS-level Further Mathematics, and only 4% of those with a grade A at GCSE did (Noyes & Adkins, 2016).

Our findings show an association between student participation in Further Mathematics and having a positive attitude to mathematics, the support of their parents or teachers, and confidence in their own mathematical abilities. This echoes the previous findings of Tanner, Lyakhova & Neate (2016). However, there is also evidence that there is a stigma attached to Further Mathematics as a subject only suitable for very able students and only required by the most prestigious universities. Such perceptions may be exacerbated by the perceived difficult nature of Further Mathematics which may be related to how Further Mathematics is taught (such as classes being fast-paced, taught across several school settings, not offering enough contact hours or being offered through FMSP blended learning model) rather than the difficulty of the material. It has been argued (Smith, 2010b) that re-establishing Further Mathematics as a course for everyone who wants to develop their mathematical skills, may help to encourage more girls (as well as boys) to take the subject. One may further argue that students with lower self-beliefs in their mathematical abilities but who are planning to study for a mathematics degree, would benefit the most from taking Further Mathematics. As we have seen in this study, studying Further Mathematics may help some students feel safer once they are at university in terms of feeling better able to cope with their courses. For this reason, we would recommend that teachers prompt a conversation about studying Further Mathematics with any student considering studying A-level Mathematics.

There remain numerous challenges as well as the question of whether Further Mathematics is the best possible preparation for a mathematics degree.

While the students involved in the interviews who took Further Mathematics would not change their choice, we have seen that those who did not take Further Mathematics were often good at defending their decision. This may suggest that intrinsic motivation played a more significant role than we were able to discover (Zimmerman, 1990) and more questions on this matter may need be included in surveys such as ours. Furthermore, we felt that students used different strategies for making their A-level choice which depended on whether they felt resolved or unresolved about their future degree choices which also was not part of the survey. Clearly, taking Further Mathematics meant keeping their options open for some and restricting choices for others. Thus, caution is needed when advising students to take more mathematics at the expense of a breadth of subjects. Some students interviewed in this study identified a perceived benefit from taking a diverse range of subjects, particularly in terms of studying essay based subjects alongside mathematics (see also (Cleaves, 2005)).

There are concerns that the latest reforms to the structure of A-levels in England and Wales could have a significant impact on the uptake of Further Mathematics (Lee et al., 2016; Smith 2017). The new mathematics A-levels now have a fully prescribed curriculum with emphasis placed on proof and how to construct a logical argument, problem solving, and modelling (Baldwin & Lee, 2017), reforms which have been broadly welcomed. In England, the new A-levels will be linear in structure with final examinations testing the entire two-year course. Consequently, the AS-level will become a separate qualification no longer available as an exit qualification half way through an A-level course. This shift is thought to allow teachers to accentuate the interconnections of mathematical topics, but the authors share concerns that have been expressed previously that the separation of the AS and A-levels may affect the uptake of Further Mathematics (Smith, 2017) and, more generally, disadvantage those students who may need to explore more subjects to clarify their career and degree choices. Moreover, the funding arrangements for post-16 education currently in place in England are perceived as actively discouraging schools and colleges from allowing students to take a fourth A-level (Kewin & Donhowe, 2017) which could also have a significant impact on Further Mathematics uptake (Smith, 2017; MEI, 2016).

Studies on mathematics transition internationally often stress that changes in pedagogy and syllabus at school level are needed to tackle problems associated with the surface-learning attitude often adopted in school (Kajander & Lovric, 2005; Liston & O'Donoghue, 2010) and the present study reinforces this message. The difficulties in adapting to university mathematics identified at interview in the present study include both the new unfamiliar abstract content and the new approach to learning mathematics with its emphasis on independent thought. Some students reported they had difficulties with the speed of university learning which assumed students could work independently to fill the gaps in their understanding when learning new material. The preference indicated by some students for memorisation over understanding highlights a reliance on the surface learning approaches, particularly amongst those studying at universities with lower entry requirements. The persistence of such thinking in transition from school to university mathematics has been characterised as a "beliefs overhang" of approaches to learning previously experienced through school as dependable by students (Daskalogianni & Simpson, 2001). The evidence in this study allows us to suggest that studying Further Mathematics does not challenge these beliefs amongst some students and so may not offer the best possible preparation for studying advanced mathematics.

Johnston-Wilder & Lee (2010) argue that the development of "mathematical resilience" should be a key part of mathematical education. This concept of resilience can be seen as a combination of three factors: value (a belief in the value of mathematics), struggle (a realisation that hard work is required), and growth (a belief that everyone can succeed in mathematics), which lead to mathematical resilience as a realisation that understanding grows from struggling (Kooken et al. 2016). In particular, Johnston-Wilder & Lee (2010) highlight that some students never meet sufficient challenge in mathematics at school to require them to develop their resilience. For some, this lack of challenge can have long term consequences for student success (see for instance (Ward-Penny, Johnston-Wilder & Lee, 2011) and (Daskalogianni & Simpson, 2002)). In view of our findings, one may be encouraged to speculate that some students who could be appropriately stretched by Further Mathematics do not currently take it, while studying extension papers (such as STEP) could provide a better challenge for some current Further Mathematics students. One of the interviewed students argued how struggling through the extension papers helped him to decide if mathematics was the right choice of a degree for him: "After all of this exposure [to STEP and MAT questions], I was *still* enjoying mathematics and was interested in taking it further, so it was natural to take Mathematics to the next level". Although extension papers have been designed to assess potential-to-thrive on a selection of very demanding university degree courses, their potential for developing the mathematical abilities, resilience and confidence of students should be explored more widely (see also (Darlington, 2015b)).

Although the numbers of students involved in this study who studied with the FMSP is small, there is evidence that the FMSP blended approach to teaching is associated to a perception of an improved approach to mathematics. However, to cope with the FMSP approach, students not only need to be good at mathematics, but also need a supportive parental environment and the motivation to invest their own time and organise their own work. The recent review of post-16 mathematics (Smith, 2017) suggests that to address the capacity shortages in 16-18 mathematics provision, thoughts should be given to incorporating technology into teaching, but also highlights that more research here is needed. The blended learning of the FMSP is a fine example of technology being used, but the advantages and problems of this approach have not been fully understood. The whole experience of studying with the FMSP is risky and known to remove the feeling of being in control for students (Smith, 2010a), yet it seems that this experience may contain some valuable opportunities for students to practice becoming independent learners and develop their resilience. While it may be viewed as a value added by the FMSP courses, academic achievement is believed to be highly dependent on student self-regulation (Zimmerman, 1983; Zimmerman & Pons, 1986).

Finally, we found some anecdotal evidence that university outreach activities expose students to mathematics "beyond the syllabus" benefiting the transition from school to university mathematics (see also (Cleaves, 2005) for more evidence on high achieving students choosing A-level and future degree choices). Given that schools and colleges do not always have the capacity to offer such experiences, creating more projects aimed at post-16 students should be beneficial (see also (Smith, 2017) for further references to other projects aimed to enhance the learning of mathematics at post-16). From this point of view the recent withdrawal of government funding for the "Mathematics Underground" project is undoubtedly worrying.

In conclusion, this study highlights that Further Mathematics as taught before 2017, was perceived to be difficult but beneficial by mathematics undergraduates, yet, it may not provide the ideal preparation for transition to university. It may be that the reformed qualifications which are being introduced this year may address some of the issues we have identified and the present findings could be used as a starting point for a comparison study when the new qualifications have become established. However, we have shown here that opportunities to challenge student approaches to learning mathematics at school may already exist in the form of the blended learning provided by the FMSP, or the extension papers such as STEP. The current study shows an association between a perception of a broader understanding of mathematics and studying with the FMSP, however it does not establish if this is causal or a result of self-selection amongst students willing to pursue studying in this more challenging model. Similarly, in the case of the extension papers, we do not know if it was the nature of the questions, the relative learner independence when exploring them, or the personal qualities of the students that contributed to the different attitudes in these students that this study registered. Further research is required to identify which aspects of these approaches could help students to develop into self-regulated and resilient learners of mathematics, without reducing participation, as well as into utilising these approaches to improve transition from school to university mathematics.

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Acknowledgement:

This study was initiated with Howard Tanner (University of Wales Trinity St David). Sadly, Howard passed away before the study was completed and is greatly missed. We would like to acknowledge his contribution to the study as it could not have been completed without his expertise, encouragement and support.

We would also like to thank the anonymous referees for their helpful and insightful comments.

Appendix 1. The Questionnaire

Questions on mathematics:

I am naturally gifted at mathematics.

Hard work is more important than ability in mathematics.

I enjoy doing mathematics.

I plan to continue studying mathematics to the highest level I can.

I plan to give up studying Mathematics as soon as I can.

I have to work hard in mathematics just to keep up.

I find it easier to memorise mathematical techniques than try to understand them.

Mathematics is the subject I am best at.

I am likely to study a mathematics related course next year.

I am good at algebra.

I’m not good enough to study Mathematics at a high level.

I enjoy the certainty of mathematics as there is always an answer.

Questions on GCSE and A-level Mathematics

I found GCSE mathematics easy.

I found AS level mathematics hard.

Only gifted mathematicians should study A-level Mathematics.

I found AS level mathematics easy.

My parents thought it was important to study at least an AS-level mathematics

You should not choose an AS or A-level mathematics unless you are good at algebra

I think that having at least an AS-level in mathematics helps if you apply for University.

Teacher(s) encouraged me to study AS/A-level mathematics

Having at least an AS-level in mathematics helps to get you a job.

There was a bigger jump in difficulty from GCSE to AS-level in mathematics than for my other subjects.

Questions on A-level Further Mathematics

My parents thought it was important for me to study Further Mathematics.

Teacher(s) discouraged me from studying Further Mathematics.

Having a qualification in Further Mathematics helps you to get into a better university.

A qualification in Further Mathematics helps you to get a good job.

Only gifted mathematicians should study Further Mathematics.

You don’t need Further Mathematics to get in to a good university.

It was more important for me to study a range of subjects than to study Further Mathematics.

I wasn’t good enough to do Further Mathematics

My algebra was not good enough for Further Mathematics.

Further Mathematics requires more self-study than other subjects.

I wish I had studied Further Mathematics.

Studying Further Mathematics gave me a broader understanding of mathematics in general.

Studying Further Mathematics made the transition to University level work easier.

I would have struggled to cope with the mathematics in my course if I had not studied Further Mathematics.

There is a step up in difficulty between Mathematics and Further Mathematics.

Appendix 2. Tables of results.

Table 3. Selected significant differences (Mann-Whitney) between those who had and had not studied Further Mathematics amongst Mathematics undergraduate students.

Survey Statements	Further Maths	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	p	Z	r
I am naturally gifted at maths.	Yes	25%	59%	12%	3%	1%	138	<0.001	-3.85	-0.25
	No	13%	48%	31%	8%	0%	98			
Teacher(s) encouraged me to study AS/A-level Maths.	Yes	60%	25%	9%	4%	1%	138	0.002	-3.10	-0.20
	No	40%	37%	12%	6%	5%	98			
Teacher(s) discouraged me from studying Further Maths.	Yes	3%	9%	12%	29%	48%	138	<0.001	4.08	0.27
	No	3%	10%	29%	38%	20%	97			
My parents thought it was important for me to study Further Maths.	Yes	12%	19%	51%	12%	7%	138	<0.001	-3.85	-0.25
	No	1%	8%	59%	24%	7%	96			
Having a qualification in Further Maths helps you to get into a better university.	Yes	21%	53%	14%	10%	2%	138	<0.001	-4.02	-0.26
	No	12%	32%	35%	19%	2%	96			
A qualification in Further Maths helps you to get a good job	Yes	13%	36%	41%	9%	1%	138	0.002	-3.16	-0.21
	No	4%	28%	44%	18%	5%	95			

Table 4. Significant differences (Mann-Whitney) between genders amongst Mathematics undergraduate students.

Survey Statements	Gender	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	p	Z	r
a) Attitudes to maths and Further Maths:										
All Students:										
A qualification in Further Maths helps you to get a good job	M	11%	38%	40%	7%	4%	135	0.001	3.25	0.22
	F	6%	23%	47%	22%	2%	87			
I am good at algebra.	M	33%	58%	9%	1%	0%	137	0.005	2.79	0.19
	F	20%	59%	19%	2%	0%	86			
Students who had not studied Further Maths:										
I wasn't good enough to do Further Maths	M	0%	2%	27%	53%	18%	60	0.006	-2.76	-0.29
	F	0%	6%	50%	38%	6%	32			
b) Considering transition to and studying at university:										
Students who had studied Further Maths:										
I would have struggled to cope with the maths in my course if I had not studied Further Maths.	M	13%	32%	26%	22%	7%	76	0.023	-2.27	-0.20
	F	25%	36%	25%	15%	0%	53			

Table 5. Selected significant differences (Kruskall-Wallis) between university groups amongst Mathematics undergraduate students.

	University Group[1]	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	p
a) Attitudes to mathematics and Further Maths:								
Students who had studied Further Maths:								
Having a qualification in Further Maths helps you to get into a better university.	1	34%	48%	12%	6%	0%	50	0.017
	2	17%	52%	17%	14%	2%	66	
	3	5%	68%	9%	9%	9%	22	
b) Considering transition to and studying at university:								
All Students:								
I find it easier to memorise mathematical techniques than try to understand them.	1	7%	17%	22%	37%	17%	54	0.003
	2	3%	22%	32%	32%	10%	121	
	3	17%	32%	20%	28%	3%	60	
Students who had studied Further Maths:								
I would have struggled to cope with the mathematics in my course if I had not studied Further Maths	1	34%	36%	18%	12%	0%	50	<0.001
	2	13%	28%	36%	20%	3%	64	
	3	5%	36%	14%	32%	14%	22	

[1]University Groups as defined in Table 1 for mean UCAS points on entry Group 1:>480, Group 2: 340-480, Group 3: <340.

Table 6. Selected significant differences (Mann-Whitney) between those who studied Further Mathematics using the FMSP and those who studied it in school amongst STEM undergraduate students.

Questions	FMSP	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	p	Z	r
I enjoy doing maths	Yes	83%	11%	6%	0%	0%	18	0.008	2.66	0.20
	No	47%	48%	2%	2%	1%	154			
Maths is the subject I am best at.	Yes	89%	6%	6%	0%	0%	18	0.004	2.90	0.22
	No	51%	33%	10%	5%	1%	154			
My parents thought it was important for me to study Further Maths.	Yes	28%	22%	50%	0%	0%	18	0.006	2.75	0.21
	No	10%	16%	52%	15%	7%	155			
Studying Further Maths gave me a broader understanding of maths in general.	Yes	77%	24%	0%	0%	0%	17	0.001	3.27	0.25
	No	35%	53%	10%	2%	0%	152			